

Secondary Math 3H

Benchmark 3 Review

Name _____ Date _____ Period _____

Find the area of each triangle using the area formula or Herons formula. Show work!

1. $A = 52^\circ$, $b = 14$ m, $c = 21$ m

$$A = \frac{1}{2}(14)(21)\sin(52^\circ) = 115.84 \text{ m}^2$$

$$A = \frac{1}{2}ab\sin C^\circ$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

2. $C = 112^\circ$, $a = 1.8$ in, $b = 5.1$ in

$$A = \frac{1}{2}(1.8)(5.1)\sin(112^\circ) = 4.26 \text{ in}^2$$

3. $a = 5$ cm, $b = 9$ cm, $c = 7$ cm

$$s = \frac{5+9+7}{2} = 10.5 \quad A = \sqrt{10.5(10.5-5)(10.5-9)(10.5-7)} = 17.41 \text{ cm}^2$$

4. $a = 19.3$ in, $b = 22.5$ in, $c = 31$ in

$$s = \frac{19.3+22.5+31}{2} = 36.4$$

$$A = \sqrt{36.4(36.4-19.3)(36.4-22.5)(36.4-31)} = 216.1 \text{ in}^2$$

Use the Law of Sines to solve the triangle. Show work!

5. $A = 50^\circ$, $B = 62^\circ$, $a = 4$

$$\frac{\sin 50^\circ}{4} = \frac{\sin 62^\circ}{b}$$

$$b = 4.6, \angle C = 68^\circ, c = 4.8$$

$$\frac{\sin 68^\circ}{c} = \frac{\sin 50^\circ}{4}$$

6. $B = 16^\circ$, $C = 103^\circ$, $c = 12$

$$\frac{\sin 103^\circ}{12} = \frac{\sin 16^\circ}{b}$$

$$b = 3.4, \angle A = 61^\circ, a = 10.8$$

$$\frac{\sin 61^\circ}{a} = \frac{\sin 103^\circ}{12}$$

7. $A = 49^\circ$, $a = 32$, $b = 28$

$$\frac{\sin 49^\circ}{32} = \frac{\sin B}{28}$$

$$\angle B = 41.3^\circ, \angle C = 89.7^\circ, c = 42.4$$

$$\frac{\sin 89.7^\circ}{c} = \frac{\sin 49^\circ}{32}$$

8. $C = 103^\circ$, $b = 46$, $c = 61$

$$\frac{\sin 103^\circ}{61} = \frac{\sin B}{46}$$

$$\angle B = 47.3^\circ, \angle A = 29.7^\circ, a = 31$$

$$\frac{\sin 29.7^\circ}{a} = \frac{\sin 103^\circ}{61}$$

Use the Law of Cosines to solve each triangle. Show work!

9. $B = 35^\circ$, $a = 43$, $c = 19$

$$b^2 = 43^2 + 19^2 - 2(43)(19)\cos 35^\circ$$

$$b = 29.5$$

$$\frac{\sin 35^\circ}{29.5} = \frac{\sin C}{19}$$

$$\angle C = 21.7^\circ$$

$$\angle A = 123.3^\circ$$

10. $b = 22$, $c = 31$, $A = 82^\circ$

$$a^2 = 22^2 + 31^2 - 2(22)(31)\cos 82^\circ$$

$$a = 33.4$$

$$\frac{\sin B}{22} = \frac{\sin 82^\circ}{33.4}$$

$$\angle B = 37.9^\circ, \angle C = 60.1^\circ$$

Convert each angle from degrees to radians. Show work!

$$11. 150^\circ \cdot \frac{180^\circ}{1} \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{6}$$

$$12. 75^\circ \cdot \frac{180^\circ}{1} \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{12}$$

$$13. 240^\circ \cdot \frac{180^\circ}{1} \cdot \frac{\pi}{180^\circ} = \frac{4\pi}{3}$$

$$14. 310^\circ \cdot \frac{180^\circ}{1} \cdot \frac{\pi}{180^\circ} = \frac{31\pi}{18}$$

Convert each angle from radians to degrees. Show work!

$$15. \frac{\pi}{5} \cdot \frac{180^\circ}{\pi} = 36^\circ$$

$$16. \frac{3\pi}{8} \cdot \frac{180^\circ}{\pi} = 67.5^\circ$$

$$17. \frac{12\pi}{5} \cdot \frac{180^\circ}{\pi} = 432^\circ$$

$$18. 0.235 \cdot \frac{180^\circ}{\pi} = 13.46^\circ$$

Find the exact value of each expression. (No decimals!)

$$19. \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$20. \sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$21. \tan \frac{\pi}{3} = \sqrt{3}$$

$$22. \csc \frac{2\pi}{3} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

23. Low tide is at 9:12 am and high tide is at 3:12 pm. The water level varies 48 inches between low and high tide. Write a cosine function to represent the change in water level. Show work!

$$f(t) = a \cos \omega t \quad a = -24 \quad F = 12 \text{ hrs.} \quad \omega = \frac{2\pi}{12} = \frac{\pi}{6}$$

$$f(t) = -24 \cos\left(\frac{\pi}{6}t\right)$$

24. Use symmetry to find the exact values of $\sin \theta$ and $\cos \theta$, for a) $\theta = -\pi/3$, b) $\theta = -5\pi/6$

$$a) \sin \theta = -\frac{\sqrt{3}}{2}, \cos \theta = \frac{1}{2}$$

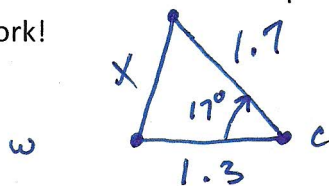
$$b) \sin \theta = -\frac{1}{2}, \cos \theta = -\frac{\sqrt{3}}{2}$$

25. Find all solutions in the interval $[0, 2\pi)$ for $6 - 8\cos x - 10 = 0$. Show all work!

$$\begin{aligned} -8\cos x - 4 &= 0 \\ -8\cos x &= 4 \\ \cos x &= -\frac{1}{2} \end{aligned}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

26. You and a friend hike 1.3 kilometers due west from a campsite. At the same time, two other friends hike 1.7 kilometers at a heading of N17°W from the campsite. To the nearest tenth of a kilometer, how far apart are the two groups? Show work!



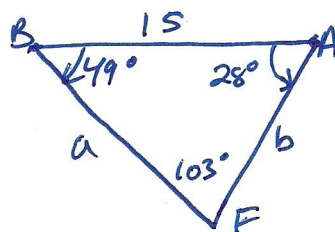
$$x^2 = (1.3)^2 + (1.7)^2 - 2(1.3)(1.7)\cos 17^\circ$$

$$x = .6 \text{ km}$$

27. Two fire-lookout stations are 15 miles apart, with station B directly west of station A. Both stations spot a fire. The bearing of the fire from station A is S28°W and the bearing of the fire from station B is S49°E. How far, to the nearest tenth of a mile, is the fire from each lookout station? Draw a model, label, and show all work!

$$\frac{\sin 103^\circ}{15} = \frac{\sin 49^\circ}{b} \quad b = 11.6 \text{ miles}$$

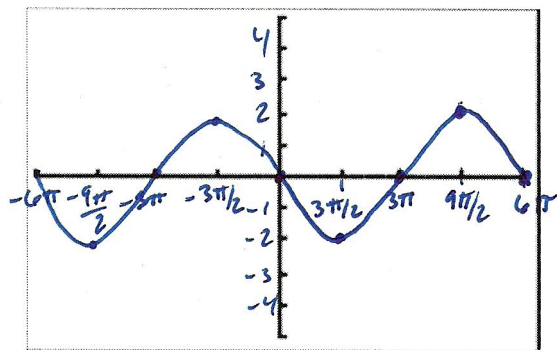
$$\frac{\sin 103^\circ}{15} = \frac{\sin 28^\circ}{a} \quad a = 7.2 \text{ miles}$$



From A: 11.6 miles
From B: 7.2 miles

Graph one period of each function. Identify the amplitude, period, phase shift, and vertical shift. Identify the y-intercept of the tangent function. Label the scale of the graphs.

28. $f(x) = -2 \sin\left(\frac{x}{3}\right) - 1$



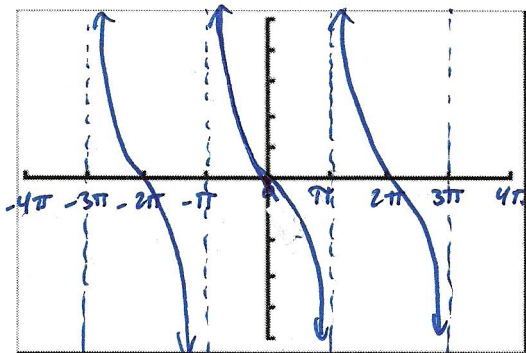
period: 6π

phase shift: none

vert. shift: down 1

Amplitude: 2

29. $f(x) = -3 \tan\left(\frac{x}{2}\right)$



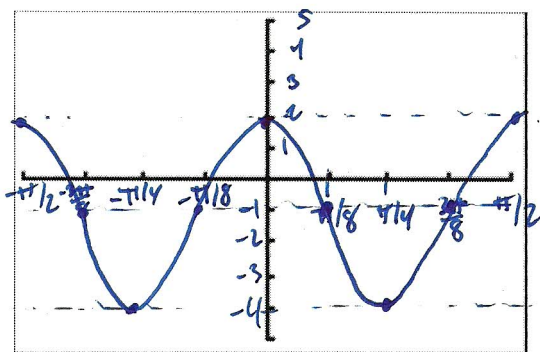
period: 2π

no phase shift

no vertical shift

y-int: (0,0)

30. $f(x) = 3 \cos(4x) - 1$



Amplitude: 3

period: $\pi/2$

phase shift: none

vertical shift: down 1

Solve.

31. $e^{2x} - 2e^x - 15 = 0$ $e^x = 5$ $e^x = -3$
 $u = e^x$ $u^2 - 2u - 15 = 0$ $x = \ln 5$
 $u^2 = e^{2x}$ $(u-5)(u+3) = 0$
 $u = 5$ $u = -3$

32. $e^{2x} - 6e^x - 8 = 0$ $e^x = 4$ $e^x = 2$
 $u^2 - 6u - 8 = 0$ $x = \ln 4$ $x = \ln 2$
 $(u-4)(u+2) = 0$
 $u = 4$ $u = -2$

33. $\log_3(4x-1) = 3$
 $3^3 = 4x-1$ $x = 7$
 $28 = 4x$
 $7 = x$

34. $\log_7(6x-5) = 1$
 $7^1 = 6x-5$
 $12 = 6x$
 $2 = x$

35. $6^{x-2} = 41$
 $\log_6 41 = x-2$
 $\log_6 41 + 2 = x$ $x \approx 4.07$
 $\frac{\log 41}{\log 6} + 2 = x$

36. $4^{2x+1} = 67$
 $\log_4 67 = 2x+1$
 $\frac{\log_4 67 - 1}{2} = x$ $x \approx 1.017$
 $\frac{\frac{\log 67}{\log 4} - 1}{2}$

Rewrite the equation in logarithmic form.

37. $6^x = 21$ $\log_6 21 = x$

38. $e^x = 15$ $\ln 15 = x$

Condense the expressions to a single logarithm.

39. $2\log x + \log y - 5\log z$
 $\log\left(\frac{x^2 y}{z^5}\right)$

40. $4\log_3 x - 2\log_3 y + \log_3 z$
 $\log_3\left(\frac{x^4}{y^2 z}\right)$

Use the properties of logarithms to evaluate each expression.

41. $\log_5 5^4 = 4$

42. $\ln e^{-6} = -6$

Find the value of each expression. Round to the nearest hundredth.

43. $\log_8 265$ $\frac{\log 265}{\log 8} \approx 2.68$

44. $\log 529$ 2.72

Find the inverse of each function.

45. $f(x) = \log(x+2) - 9$
 $x = \log(y+2) - 9$
 $x+9 = \log(y+2)$
 $10^{x+9} = y+2$
 $f^{-1}(x) = 10^{x+9} - 2$

46. $f(x) = 3 \cdot 4^{x-1} + 7$
 $x = 3 \cdot 4^{y-1} + 7$
 $\frac{x-7}{3} = 4^{y-1}$
 $\log_4\left(\frac{x-7}{3}\right) = y-1$
 $f^{-1}(x) = \log_4\left(\frac{x-7}{3}\right) + 1$